An exploration into **Cooling Process**

Internal Assessment: Mathematics Analysis and Approaches Standard Level

Formulating a Suitable Equation to Model the Cooling Process of Hibiscus Tea

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Introduction

I was inspired to undertake this investigation because of a habit which has become part of my evening routine. After finishing my homework, I usually brew hibiscus tea and drink it while reading books. The bright red colour and sour taste makes the drink especially enjoyable, but I have a repeated problem: I often begin another task and forget the cup beside me. When I return, it has either cooled into the comfortable range or become too cold. The difference between these outcomes is only several minutes, so I wanted a way of predicting the useful drinking time instead of testing the tea each evening again and again.

This situation connected naturally to functions and calculus. Temperature is continuous, it changes at a non-constant rate, and the surrounding room places a lower restriction on the cooling process. I therefore decided to measure the temperature of one preparation of hibiscus tea repeatedly and then formulate an equation which models its cooling. My research question is: How accurately can an exponential model predict the time required for say 300 mL of hibiscus tea to cool below 45°C in a room with an ambient temperature of approximately 22.4°C?

The boundary of 45°C is personally relevant because I have checked and below this value the drink tastes flat and I normally reheat it. Research about hot beverages often reports preferred temperatures close to 54-60°C, although preference changes between people and beverages (Brown and Diller, 2008). I used a lower boundary because hibiscus tea is less viscous than coffee and, in my experience, still feels warm at 45°C. This decision is subjective, but it allows the mathematics to answer a clear and useful question.

From chemistry and physics, I expected the difference between the tea temperature and the room temperature to somehow influence the cooling rate. A larger difference should cause a faster loss of thermal energy, while a smaller difference should slow the process. The liquid should not cool below the room temperature because at that point the tea and the environment have reached thermal equillibrium. Evaporation, conduction through the mug and convection at the surface all contribute to this behaviour. I anticipated an exponential relationship with a horizontal asymptote at the ambient temperature.

Background and Data Collection Methodology

Hibiscus beverages may be prepared by hot or cold extraction. Ramirez-Rodrigues et al. compared both methods and showed that hot extraction at 90°C can obtain similar anthocyanin concentrations much faster than cold extraction. For consistency, I prepared the tea using water from the same kettle and the same packet of dried Hibiscus sabdariffa calyces in every trial. This gave the exploration a real context while keeping the mathematical variable of interest as temperature over time.

For each trial, 6.0 g of dried hibiscus calyces was placed in a ceramic mug. I heated water to 90°C, measured 300 mL with a graduated cylinder, poured it into the mug, and stirred for 10 seconds. After an extraction time of four minutes the calyces were removed using the same metal strainer. A digital probe thermometer was placed approximately 3 cm below the surface without touching the mug. The first reading was defined as $t = 0$, and a temperature was recorded every 5 minutes for one hour. The experiment was repeated three times on the same desk.

The room temperature was measured before each trial and treated as 22.4°C. A fluctuation of about ±0.3°C was visible during the experiment, but this was ignored in the later model so that a single horizontal asymptote could be used. The same mug, water volume, calyx mass, stirring time, probe depth and desk position were held constant. Windows were closed and no fan was used. The thermometer displayed values to 0.1°C and the measuring cylinder had 5 mL divisions.

Using three trials should eliminate random errors and produce data which are more representative than one observation. It was not possible to control the exact initial tea temperature, the humidity, or the small amount of water remaining on the strainer. I also assumed that the thermometer did not significantly cool the liquid. These limitations were considered acceptable because the purpose was to construct a usable mathematical model rather than a full thermodynamic description.

Variables

- Independent variable: elapsed time, t, measured in minutes.
- Dependent variable: temperature of the tea, T, measured in degrees Celsius.
- Controlled variables: mug, liquid volume, hibiscus mass, extraction time, probe depth, stirring and room location.

Raw Data

Table 1 - Temperature over time from Trial 1

Time (min)	0	5	10	15	20	25	30	35	40	45	50	55	60
Temp. (°C)	78.9	69.9	62.9	55.2	49.9	44.0	38.6	34.9	31.3	28.0	27.1	26.4	25.5

Table 1.1 - Temperature over time from Trial 2

Time (min)	0	5	10	15	20	25	30	35	40	45	50	55	60
Temp. (°C)	78.2	70.7	62.3	55.8	49.5	43.5	39.1	34.5	31.8	28.4	26.7	25.9	26.1

Table 1.2 - Temperature over time from Trial 3

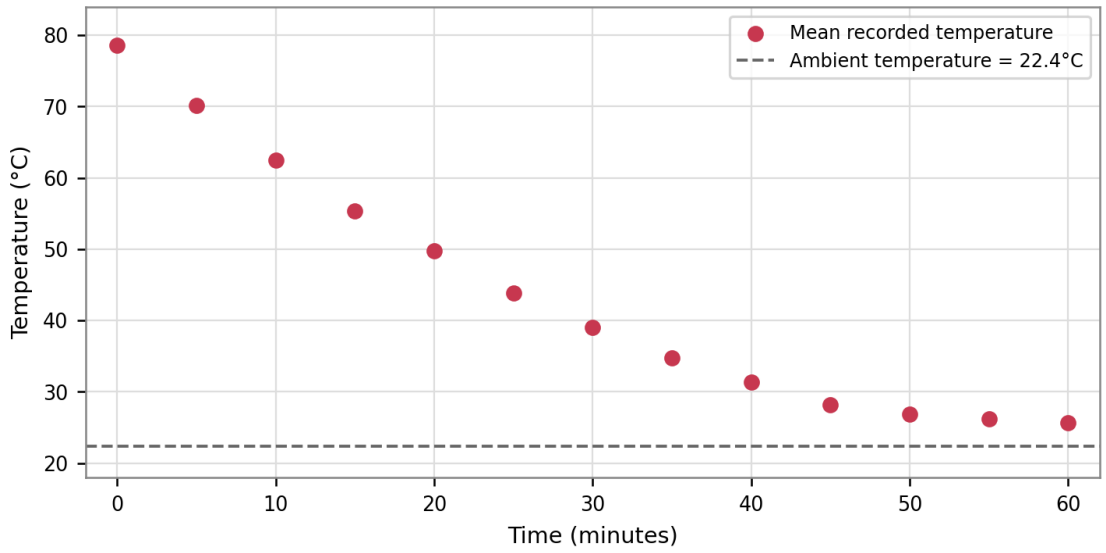
Time (min)	0	5	10	15	20	25	30	35	40	45	50	55	60
Temp. (°C)	78.7	70.0	62.3	55.2	50.0	43.9	39.2	35.0	31.1	28.2	26.9	26.3	25.5

Processed Data

Table 1.3 - Mean temperature over time

Time (min)	0	5	10	15	20	25	30	35	40	45	50	55	60
Temp. (°C)	78.6	70.2	62.5	55.4	49.8	43.8	39.0	34.8	31.4	28.2	26.9	26.2	25.7

Figure 1 - Mean temperature plotted against time for a cup of hibiscus tea



A horizontal asymptote at t = 22.4 was added to Figure 1.0 to represent the measured ambient temperature. According to Boyle's gas law, a substance cannot decrease its temperature below that of its environment. This explains why the data approaches 22.4°C rather than continuing downwards in a straight line.

The graph shows a negative correlation between temperature and time, together with a curved shape that resembles exponential decay. Temperature falls by 8.4°C in the first five minutes, but only by 0.5°C in the final five minutes. The changing differences imply that a linear equation would not be suitable for the full domain. Also, the y-intercept represents the measured initial temperature, so when t = 0, T = 78.6°C.

Producing a Suitable Equation - Attempt A

I began by using the appearance of Figure 1.0 rather than applying a known physical law. Since temperature T was plotted against time t, I treated T as y and t as x. An exponential curve with a negative slope can first be represented by raising e to the power of negative time:

$$T(t) = e^{-t}$$

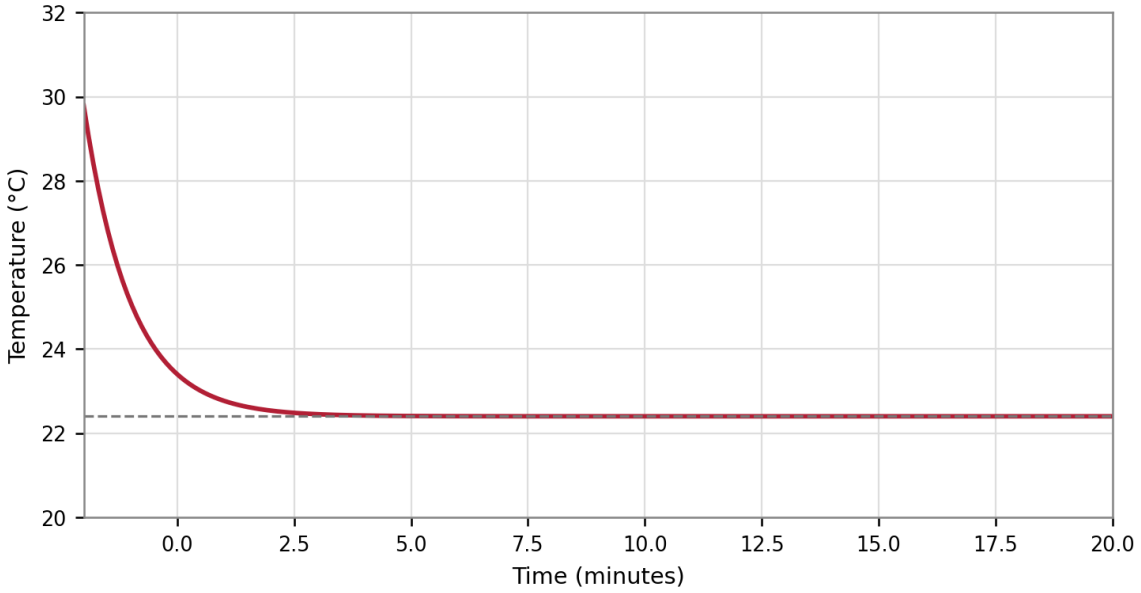
The entire graph has to be translated upward by the ambient temperature. This creates the translation constant T_a:

$$T(t) = e^{-t} + T_a$$

$$T(t) = e^{-t} + 22.4$$

This first expression includes the correct horizontal asymptote, but it still has an initial value of only 23.4°C. I graphed it to understand which transformations were missing before calculating parameters from the experimental values.

Figure 1.1 - The preliminary model $T(t) = e^{-t} + 22.4$



Although an exponential relationship is present and the curve approaches the correct asymptote, this graph does not resemble the observed cooling process. Almost all the change occurs close to t = 0. For the positive values of time used in my experiment, the function immediately becomes indistinguishable from 22.4°C.

The function must therefore be stretched horizontally so that the cooling takes place across sixty minutes. It must also be stretched vertically so that its y-intercept is close to 78.6°C. Let k represent the horizontal stretch factor and m represent the vertical stretch factor. A more useful general expression is:

$$T(t) = me^{-kt} + T_a$$

$$T(t) = me^{-kt} + 22.4$$

The constant m controls the distance between the initial temperature and the asymptote, while k controls the speed of cooling. Both constants should be positive when the negative sign is already included in the exponent. A larger k would make the tea cool more quickly. Values for these constants can be found by using two points from Table 1.3.

For Attempt A, I selected the first and last mean observations. These points cover the entire experimental interval and therefore should create the most representative model. This method does not use the intermediate points directly, which is one weakness, but the two endpoints provide two equations for the two unknown constants.

Determining the Constants for Attempt A

Beginning with the transformed exponential equation:

$$T - T_a = me^{-kt}$$

Natural logarithms can be used to remove the exponential term:

$$\ln(T - T_a) = \ln(m) - kt$$

Substituting the first point (0, 78.6) and the last point (60, 25.7) gives the following simultaneous equations. All values are kept unsimplified instead of being rounded to three significant figures, in order to ensure the utmost accuracy of the model.

$$\ln(78.6 - 22.4) = \ln(m) - 0k$$

$$\ln(25.7 - 22.4) = \ln(m) - 60k$$

Table 1.4 - Simultaneous equations used to determine m and k

Equation 1	Equation 2
ln(56.2) = ln(m)	ln3.3 = ln(m) − 60k
ln(m) = 4.02892	1.19392 = ln(m) − 60k

Subtracting Equation 1 from Equation 2 eliminates ln(m):

$$-60k = \ln(3.3) - \ln(56.2)$$

$$k = 0.047250$$

The value of m follows directly from Equation 1:

$$m = e^{\ln(56.2)} = 56.2000$$

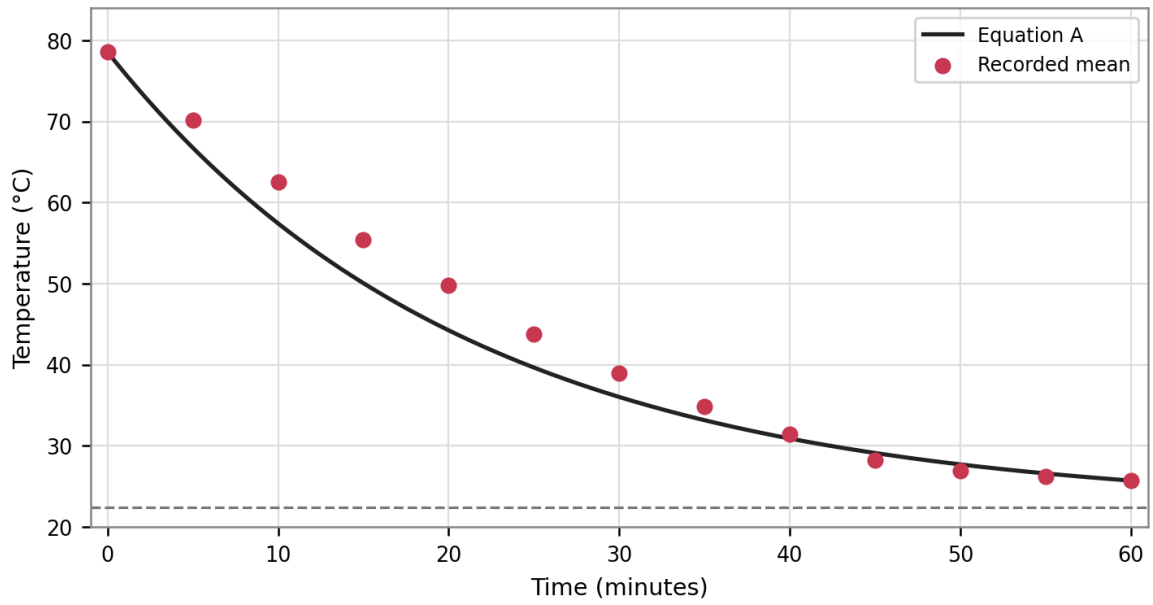
Equation A

Substituting m = 56.2000, k = 0.047250 and T_a = 22.4 into the general expression produces Equation A:

$$T(t) = 56.2000e^{-0.047250t} + 22.4 \qquad \text{Equation A}$$

The equation returns T(0) = 78.6°C and T(60) = 25.7°C, so the two points used in its construction are exact. Because it contains a negative exponential, it also approaches 22.4°C as t becomes large. These features agree with the principal observations from Figure 1.0.

Figure 1.2 - Equation A compared with the collected mean temperatures



The black curve follows the general trend of the red data points. However, Equation A underpredicts several observations between 10 and 35 minutes. It only fits the endpoints perfectly because those were the points selected for the simultaneous equations. The deviation may be caused by small changes in air change, the thermal mass of the mug or the rate of evaporation.

Real cooling data cannot be expected to follow a mathematical function exactly. Nevertheless, the repeated pattern of residuals suggests more than random error. A method which uses every transformed data point may create a more reliable value for k and reduce the influence of one anomalous measurement.

Producing a Suitable Equation - Attempt B

In Attempt B I linearised the collected data. When a relationship is linear, the gradient and intercept can be estimated using all observations rather than only two. This should account better for natural fluctuations and makes it easier to judge whether an exponential relationship is appropriate.

Starting from the same general expression:

$$T - T_a = me^{-kt}$$

$$\ln(T - T_a) = \ln(m) - kt$$

This has the linear form y = mx + c:

$$\ln(T - T_a) = (-k)t + \ln(m)$$

$$y = \ln(T - T_a), \quad mx = kt, \quad c = \ln(m)$$

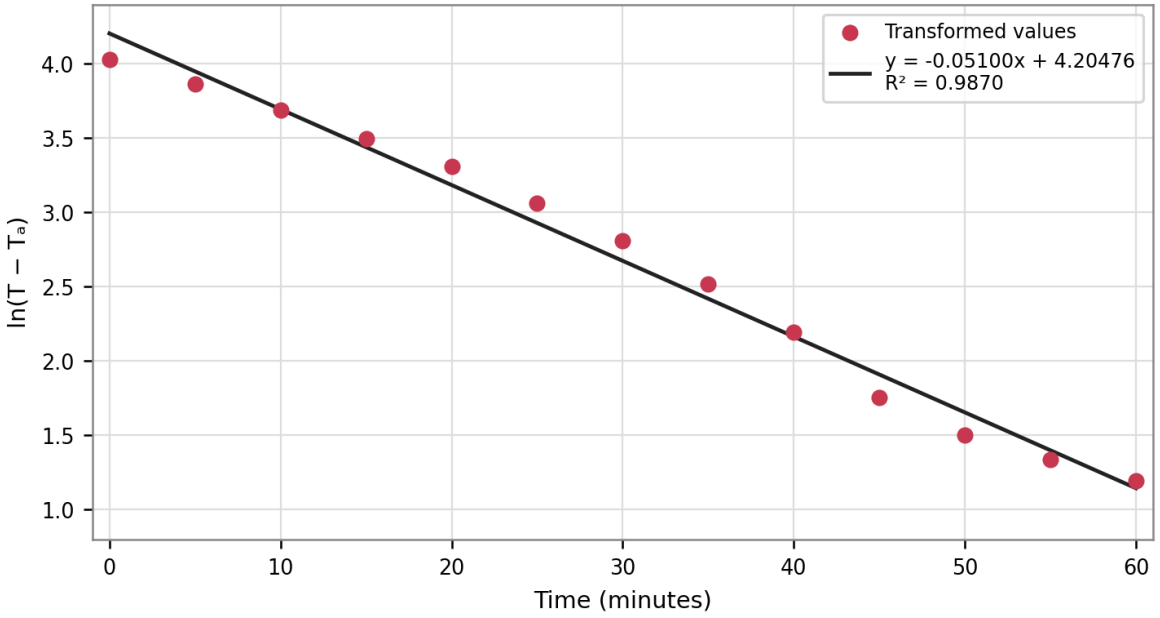
Therefore, I plotted ln(T − T_a) on the y-axis against time on the x-axis. If the points are approximately linear, the original temperature data can be treated as exponential. The processed logarithms are shown below. Because logarithms have no units, the transformed values are recorded without °C.

Table 1.5 - Values of ln(T − T_a) plotted against time

Time (s)	0	5	10	15	20	25	30	35	40	45	50	55	60
ln(T − T_a)	4.029	3.867	3.691	3.497	3.311	3.063	2.809	2.518	2.197	1.758	1.504	1.335	1.194

The use of a mean ambient temperature is important here. If T_a were changed for every row, the logarithmic transformation would no longer describe one function. I therefore maintained T_a = 22.4°C even though the room thermometer varied slightly.

Figure 1.3 - Linearised graph of ln(T − T_a) plotted against time



The transformed points are distributed closely around the line of best fit, though the later points curve below the line. The coefficient of determination is approximately 0.9871, meaning that 98,71% of the variation in temperature is explained by time. This high value supports the use of a exponential equation.

From the regression output:

$$y = -0.050997x + 4.20430$$

Comparing this with ln(T − T_a) = −kt + ln(m), the gradient equals −k and the intercept equals ln(m). Therefore:

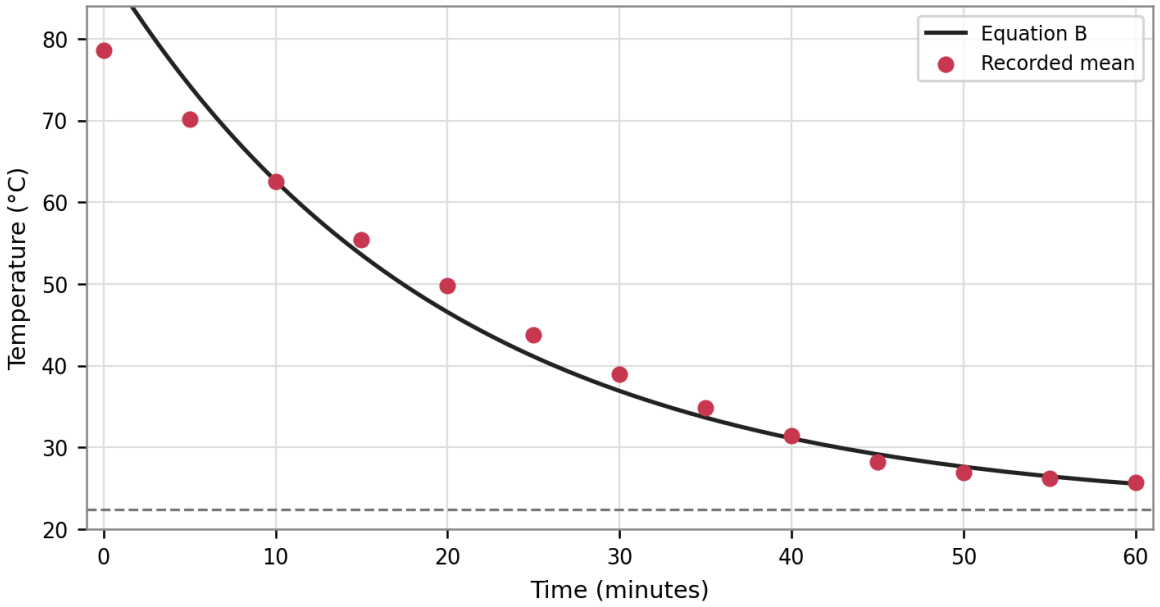
$$k = 0.050997, \qquad m = e^{4.20430} = 66.9735$$

Substituting both constants gives the second model:

$$T(t) = 66.9735e^{-0.050997t} + 22.4 \qquad \text{Equation B}$$

Equation B does not pass exactly through the first or last data point because the regression considers all thirteen values. Its y-intercept is 89.37°C, which is higher than the recorded starting temperature. This is an unexpected outcome of the line of best fit and will be considered when the models are compared.

Figure 1.4 - Equation B compared with the collected mean temperatures



Equation B follows the data during most of the monitored interval, but it overpredicts the temperature at t = 0 and underpredicts some temperatures around 20-35 minutes. The deviation in the beginning is greater than for Equation A. In the final twenty minutes, however, the curve is relatively close to the observations.

The linearisaton uses every point, but the logarithmic transformation changes the influence of errors. A difference of 1°C near the ambient temperature produces a much larger proportional change in ln(T − T_a) than the same error at a high temperature. Consequently, fitting the transformed values does not necessarily minimise the errors measured in degrees Celsius.

Producing a Suitable Equation - Attempt C

Since both equations were created mainly from the shape of the experimental graph, I completed additional research. Newton's Law of Cooling, first investigated in the early eighteenth century, states that the rate of heat loss of a body is directly proportional to the temperature difference between the body and its environment (OpenStax, 2021). In mathematical notation:

$$\frac{dT}{dt} \propto (T - T_a)$$

Because the hibiscus tea is cooling, the instantaneous change in temperature must be negative. Introducing a positive constant k gives:

$$\frac{dT}{dt} = -k(T - T_a)$$

Solving the Differential Equation:

The variables temperature and time can be seperated:

$$\frac{1}{T - T_a}\, dT = -k\, dt$$

Integrating both sides gives:

$$\int \frac{1}{T - T_a}\, dT = \int\, -k\, dt$$

$$\ln|T - T_a| = -kt + C$$

Taking e to the power of both sides removes the logarithm:

$$T - T_a = e^{-kt+C} = e^{C} e^{-kt}$$

The value e^C is another positive arbitrary constant, which I labelled Z. Rewriting the solution with temperature as a function of time gives:

$$T(t) = Ze^{-kt} + T_a$$

where T is temperature in degrees celsius, t is time in minutes, T_a is the ambient temperature, k is the cooling coefficient and Z is a constant set by an initial condition. For this experiment the room temperature is inserted:

$$T(t) = Ze^{-kt} + 22.4$$

The derived solution has the same form as my earlier equations, but it is now justified by a differential equation. Newtons law assumes a constant surrounding temperature and a rate coefficient which does not change during the process. These assumptions simplify the combined effects of convection, conduction, radiation and evaporation into one constant.

Determining the Constants for Attempt C

To determine Z and k, I chose two intermediate observations rather than the endpoints. I used T = 55.4°C at t = 15 minutes and T = 28.2°C at t = 45 minutes. The first point is after the steepest early cooling and the second is close to the drinking threshold, so they should better represent the useful part of the curve.

$$\text{I} \quad 55.4 = Ze^{-15k} + 22.4$$

$$\text{II} \quad 28.2 = Ze^{-45k} + 22.4$$

Table 1.6 - Simultaneous equations used to determine Z and k

Equation 1	Equation 2
$33.0 = Ze^{-15k}$	$5.8 = Ze^{-45k}$
$\ln(33.0) = \ln(Z) - 15k$	$\ln(5.8) = \ln(Z) - 45k$
$3.49651 = \ln(Z) - 15k$	$1.75786 = \ln(Z) - 45k$

Subtracting Equation 1 from Equation 2 removes ln(Z):

$$\ln(5.8) - \ln(33.0) = -30k$$

$$k = 0.057955$$

This value is positive, as expected for a decreasing exponential model. I retained six decimal places to prevent the final predicted time from being affected by premature rounding.

Equation C

Substituting k into Equation 1 determines Z:

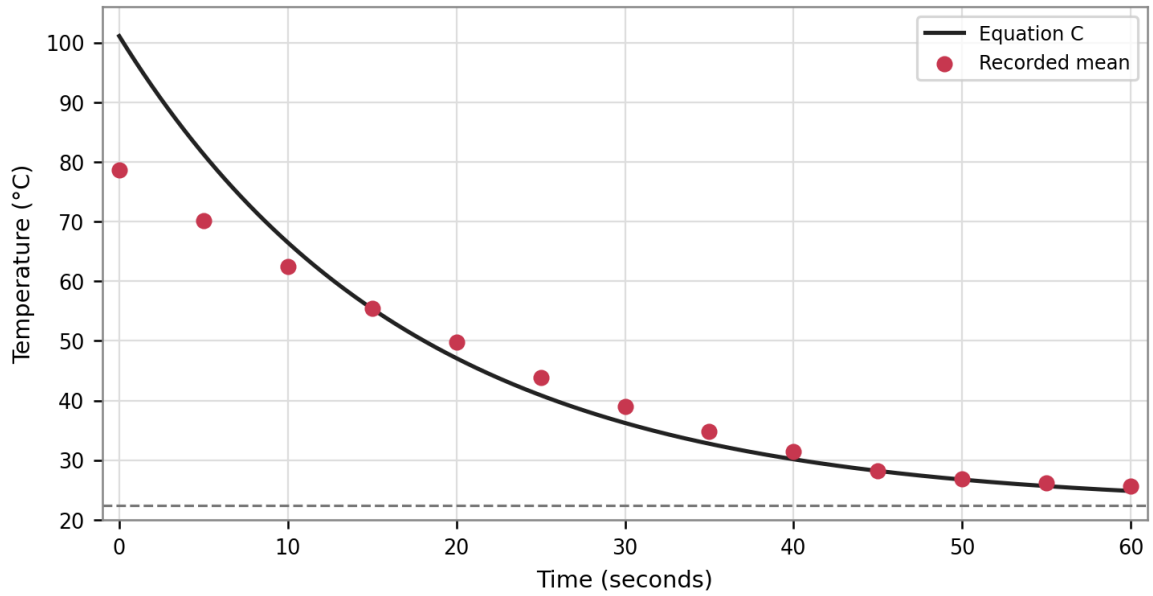
$$Z = (33.0)e^{15(0.057955)} = 78.7149$$

The third model is therefore:

$$T(t) = 78.7149e^{-0.57955t} + 22.4 \qquad \text{Equation C}$$

This equation passes exactly through the observations at 15 and 45 minutes. It also approaches the horizontal asymptote of 22.4°C, satisfying the physical limitation used throughout the exploration. However, its predicted original value is 101.11°C because T(0) = 78.7149 + 22.4. This is impossible for water prepared at 90°C and differs substantially from the recorded 78.6°C.

Figure 1.5 - Equation C compared with the collected mean temperatures



The largest discrepancy occurs at the beginning. As time increases, the curve moves closer to the data and is especially accurate around the two chosen points. The x-axis is displayed in seconds by the graphing software, although the numerical time values were entered in five-minute intervals.

Comparing All Three Developed Models

To compare the models numerically, I calculated the temperature predicted by each equation for every recorded time. More decimal places are shown for the predicted values than the thermometer could measure because they were generated directly from the equations.

Table 1.7 - Recorded and predicted temperature values

Time (min)	Recorded T (°C)	Equation A (°C)	Equation B (°C)	Equation C (°C)
0	78.6	78.6000	89.4046	101.1149
5	70.2	66.7746	74.3237	81.3127
10	62.5	57.4375	62.6371	66.4922
15	55.4	50.0650	53.5808	55.4000
20	49.8	44.2438	46.5628	47.0983
25	43.8	39.6475	41.1244	40.8850
30	39.0	36.0184	36.9101	36.2347
35	34.8	33.1528	33.6442	32.7544
40	31.4	30.8903	31.1135	30.1495
45	28.2	29.1038	29.1523	28.2000
50	26.9	27.6932	27.6325	26.7409
55	26.2	26.5794	26.4548	25.6489
60	25.7	25.7000	25.5422	24.8316

Equation A is exact at 0 and 60 minutes, Equation C is exact at 15 and 45 minutes, and Equation B is not exact at any point because it was generated through regression. Visual comparisons alone do not identify the strongest model, so I used the root mean square error.

Root Mean Square Error

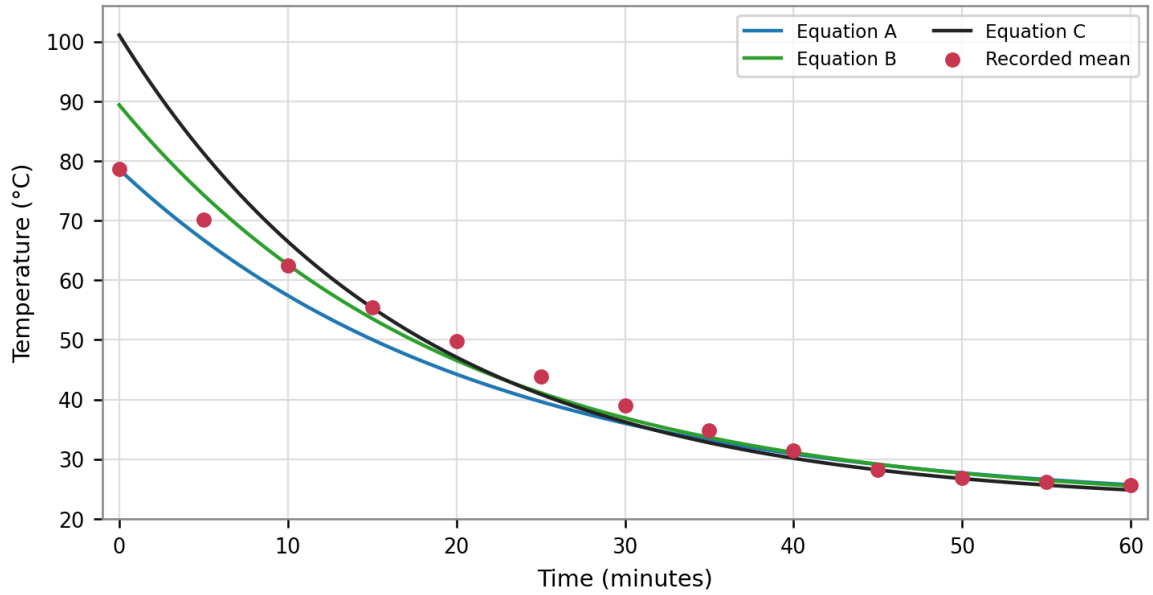
RMSE is the standard deviation of the residuals, which are the differences between the recorded temperatures and the values predicted by a model. Squaring prevents positive and negative residuals from cancelling. A lower result shows that the regression line is a better suit for the data.

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(T_i - \hat{T}_i\right)^2}$$

Table 1.8 - Root mean square error of each equation

Model	RMSE (°C)	Method used
Equation A	3.130	First and last observations
Equation B	3.530	Linear regression of logarithms
Equation C	7.214	Newton model using t = 15 and 45

Figure 1.6 - All three equations compared with the collected data



Equation A has the smallest full-data RMSE, although Equation B is similar. Equation C has the highest error because its extrapolated starting temperature is much too high. This initially suggests that Equation A is the strongest model for the entire sixty-minute interval.

Interpretation of the Model Comparison

The relative success of Equation A can be linked to its algebraic derivation. It was constructed from the first and last observed values, and its rate constant describes the overall change during the experiment. Its predictions are not exact in the middle, but the errors are spread across the full domain. The model is specific to this mug, volume and room, so the same constants should not be assumed for another cooling process.

Equation B also produces a reasonable RMSE. The linearisation uses every point and reduces the influence of high temperature residuals, but the transformed regression has a higher intercept than the actual data. The value $R^2 = 0.9871$ seems very high, yet this statistic applies to $\ln(T - T_a)$, not to temperature itself. It is possible for a line to fit the transformed graph well while producing larger errors after transforming back.

Equation C is the least accurate if all readings are included. Its main flaw is the assumption that the room remained exactly 22.4°C. As the tea cooled, the air immediately surrounding the mug would of increased in temperature. The difference between the tea and its environment therefore became smaller than the model assumed, slowing the real cooling process. The mug also stored heat and returned part of it to the liquid, which is not represented by a single constant k.

On closer observation, the underprediction and overprediction from Equation C are concentrated at the start. The research question is not concerned with the initial temperature; it asks when the tea falls below 45°C. This happens after about twenty minutes. Therefore, the early error may be less important for the practical purpose even though it increases the overall RMSE considerably.

To test this, I removed the predicted and recorded values from the first ten minutes and recalculated the RMSE of Equation C. This adapted comparison begins at $t = 10$ and keeps the data near the drinking threshold. Removing these values is justified because I would never drink the tea immediately after brewing.

Adapted Comparison of Equation C

Figure 1.7 - Equation C compared with the data after disregarding the first ten minutes

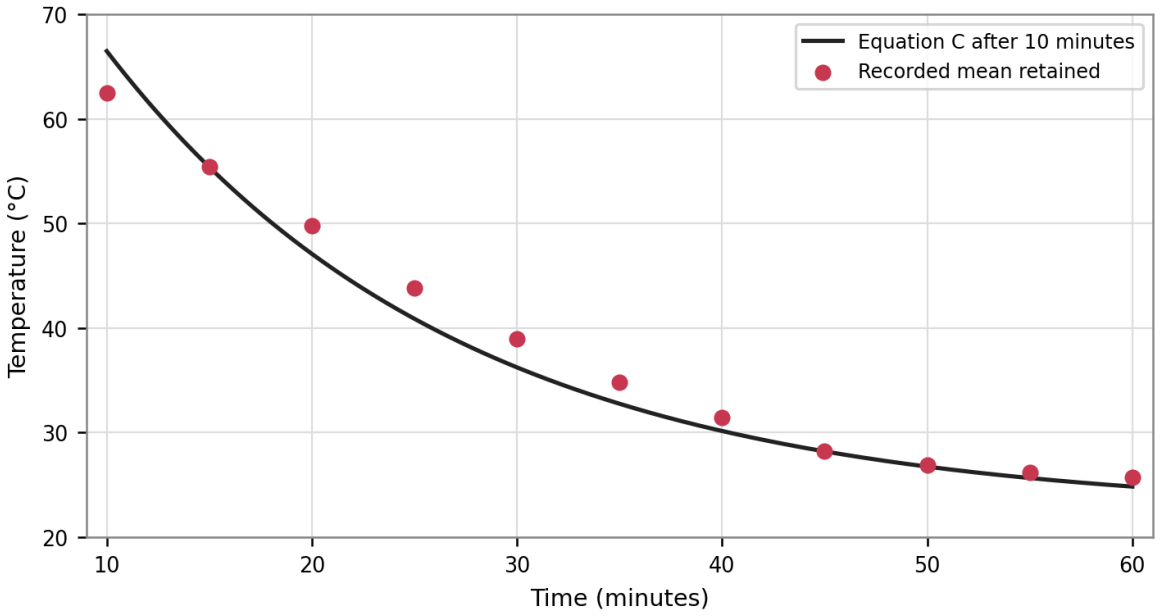


Table 1.9 - Adapted RMSE for Equation C

Comparison	RMSE (°C)
Equation C, all values	7.214
Equation C, t ≥ 10 minutes	1.991

After disregarding the first ten minutes, the calculated RMSE decreases from 7.214 to 1.991°C. This is smaller than the RMSE of Equations A and B calculated over the whole interval. The improvement shows that Equation C becomes more accurate at higher values of time, especially once the tea is close to the lower end of its preferred temperature range.

The comparison is not completely equal because Equation C is now tested with fewer data points than Equations A and B. Nevertheless, it answers the investigation more directly. I therefore selected Equation C as the most suitable equation for predicting when the tea is no longer acceptably hot. It is based on a physical law, fits the later measurements, and contains the required ambient-temperature asymptote.

The adapted graph still contains residuals of approximately 2-3°C between 20 and 35 minutes. A prediction near 45°C can consequently be incorrect by several minutes. This limitation should be remembered when a single maximum waiting time is calculated.

Answering the Original Question

The original question asks for the time required for the hibiscus tea to cool below 45°C. I used Equation C and set T = 45:

$$45 = 78.7149e^{-0.057955t} + 22.4$$

$$\frac{45 - 22.4}{78.7149} = e^{-0.057955t}$$

$$\ln\left(\frac{22.6}{78.7149}\right) = -0.057955t$$

$$t = \frac{-1.2480}{-0.057955} = 21.53$$

$$t \approx 22 \text{ minutes}$$

I can therefore conclude that approximately 22 minutes is the maximum amount of time which can elapse before my hibiscus tea is no longer considered hot. This agrees with the raw data because the temperature is 49.8°C at 20 minutes and 43.8°C at 25 minutes. Linear interpolation between these readings would place 45°C at around 24 minutes, which is reasonably close to the model.

If I begin drinking within twenty minutes, the model predicts the tea will remain above my chosen minimum. The result is specific to the 300 mL volume and ceramic mug, so it should not be treated as a universal value for hibiscus tea. Even so, the investigation has transformed an ordinary routine into a practical use of exponential functions, logarithms and differential equations.

Reflection

The mathematics developed from a visual conjecture into a physically motivated model. Attempt A demonstrated transformations of e^{-t}, Attempt B used linear regression, and Attempt C connected the same form to a differential equation. Comparing the models with RMSE made the final choice less dependent on appearance alone, although the adapted comparison was necessary for the research question.

Limitations and Possible Improvements

The most significant limitation is the treatment of ambient temperature as constant. The measured room temperature varied by approximately 0.3°C, and the air beside the mug may have been warmer than the air measured farther away. A future experiment could use a second temperature probe next to the mug and model T_a as a function of time instead of a fixed asymptote.

The five-minute sampling interval was also too large during the beginning, when temperature changed quickly. Recording every minute would create more data for the regression and make the crossing of 45°C easier to identify. More than three trials would improve the reliability of the mean and permit uncertainty intervals. I did not propagate the ±0.1°C thermometer resolution through the logarithm or the final value of t.

A second improvement would be to measure mass rather than volume because evaporation reduced the amount of liquid during the hour. Covering the cup would reduce evaporation but also change the cooling mechanism, so both covered and uncovered mugs could be compared. Different mug materials and surface areas would allow the constant k to be interpreted rather than only fitted.

The model assumes one uniform temperature throughout the tea. In reality, the liquid near the surface and the wall may be cooler than liquid in the centre. I stirred only before the first measurement because repeated stirring could increase convection, but this means the probe may not represent the mean temperature of the entire cup. A controlled magnetic stirrer at a low constant speed could make the temperature more uniform.

Finally, choosing Equation C after deleting the first ten minutes introduced bias. A better comparison would calculate the RMSE of all three models on exactly the same restricted domain or use a separate validation trial which was not involved in selecting constants. Residual plots could reveal whether the errors are random. A two-term exponential or a model with a variable heat-transfer coefficient may fit the mug and liquid separately, but this would increase complexity and perhaps reduce the clarity of the exploration.

Despite these weaknesses, the exploration produced a model that is plausible in the interval relevant to my routine. The final time of about 22 minutes should be interpreted as an estimate, not as an exact rule.

Works Cited

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All experimental temperatures and graphs in this exploration were produced for the investigation. Calculations were completed using a spreadsheet and checked with graphing software.